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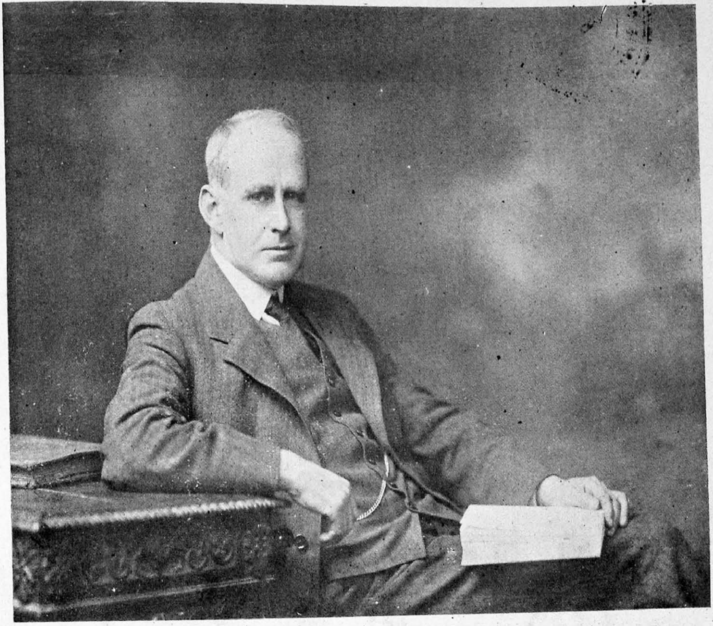

Sir Arthur Eddington

THE MATHEMATICS STUDENT

Volume V] SEPTEMBER 1937 [Number 3

OFFICE OF THE ... 8 JUN ... MADRA

ON THE REPRESENTATION OF SPHERICAL FIGURES ON A PLANE BY PARALLEL OR BY CONICAL PROJECTION

BY

A. NARASINGA RAO, *Annamalainagar.*

1. Whenever we wish to draw diagrams on paper of spherical figures, as in Astronomy, we usually rely on intuition to get a plausible figure. How uncertain a guide intuition is, may be inferred from the number of occasions on which we hesitate as to which way a line should curve, and the very unconvincing and even funny diagrams that one sees not only in answer-papers but also in printed text-books in Astronomy or Spherical Trigonometry. I had occasion recently to think over the whole question in connection with the drawing of some accurate plane representations of spherical figures and framed for my guidance certain helpful rules. These secrets I now propose to share with the readers of *The Mathematics Student.*

Conical Projection and Parallel Projection

2. There are various ways in which a 1-1 correspondence may be established between the points on a sphere and on a plane π; but since our object is to present the spherical figure as the eye sees it, we represent each point P on the sphere by the point p where OP cuts the plane, O being a fixed point which corresponds to the position of the eye. This is called *conical projection* with vertex O. When O is at infinity we have *parallel projection*, a particular case of which is *orthographic* (or *orthogonal*) projection in which every point on the sphere is imaged by its orthogonal projection on the plane. Parallel projection gives figures which are more easily drawn than general conical projection, and provides an ideal mode of representation with all the merits but without the difficulties involved in conical projection. We shall, therefore, mainly discuss the representation of figures by parallel projection. However, I give in the next paragraph, for later reference, such properties as are common to all conical and hence also to parallel projections.

Properties Common to all Projections

3. The enveloping cone of a sphere from any vertex O is always a right circular cone and its section by the plane π is a conic γ which may be taken to be an ellipse.* It will be a circle only when the plane is perpendicular to the axis of the cone. The polar plane of the vertex O with respect to the sphere cuts it in a circle Γ which projects into the conic γ on π. Hence we have

PROPOSITION I: *By conical or parallel projection, the surface of the sphere is represented in the interior of a bounding ellipse γ which becomes a circle when the join of O and the centre of the sphere is perpendicular to the plane of projection π—in particular when the projection is orthographic.*

It must be remembered that the correspondence between the spherical surface and the interior of π is not 1-1 but 2-1. In fact, the circle Γ divides the surface into two spherical caps each of which is in 1-1 correspondence with the interior of the ellipse. It is easy to see, however, that so far as the representation of plane sections is concerned, this causes no difficulty, and that each point on the image curve in π corresponds to only one point on the sphere except when the plane of the section passes through O in which case the image is a straight line segment.

Consider now a *regular curve* on the sphere, by which I mean a curve which has a tangent at each point. Its projection is also a regular curve. If the curve does not meet Γ its projection is a curve on π which does not meet the bounding ellipse γ but lies well within it. Suppose the curve meets Γ in a point P. Now the tangent plane at P passes through O since the plane of Γ is the polar plane of O, and hence the tangent to any regular curve through P projects into the intersection of π with the tangent plane at P to the sphere, i. e., into the tangent at p to γ. Hence the projection of any regular curve on the sphere which meets Γ at P is a curve on π which touches † γ at the corresponding point p.

Recollecting that every plane section of the sphere must project into a conic on π, we have

* Theoretically, it may be *any* conic. However, in all perspective representation of finite objects, we choose π so that the picture may be confined to a limited region. Not even in the restless vagaries of Modern Art has it been suggested that an object should be drawn as it would look if rays of light from the object could reach the eye from behind and the eye were incapable of distinguishing between the forward and backward directions.

† Except when the tangent at P passes through O when the projection will develop a cusp at p. For plane sections, this corresponds to the case when the projection is a line ellipse.

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PROPOSITION II: *By conical or parallel projection a circle (great or small) on the sphere projects into an ellipse on π which either does not meet γ in real points or has double contact with it.*

The bounding ellipse γ is itself the projection of a small circle Γ in the general case. All great circles on the sphere have a common centre, but their projections will not be concentric since mid-points do not project into mid-points except in parallel projection. These are some of the demerits of conical projection which will not be present in

Parallel Projection.

4. Here O is at infinity and Γ is a great circle. Hence Γ cuts every great circle, in a pair of diametrically opposite points. Since mid-points project into mid-points, we have

PROPOSITION III: *By parallel projection every great circle projects into an ellipse concentric with γ and having double contact with it at the extremities of a diameter. A small circle projects into an ellipse which lies well within γ or has double contact with it, but not at the extremities of a diameter.*

That the two contacts are at diametrically opposite points is a consequence of the two ellipses being concentric and is not an independent property.

It is worth noting that the information in Prop. III is *complete* in the sense that the projections are subject to no further restrictions. The totality of conics having double contact with a given conic γ is an ∞^3 aggregate just like the totality of small circles on a sphere. Again the aggregate of conics concentric with γ and having double contact with it have two degrees of freedom just like the great circles on a sphere. To put it differently, through three points PQR on the sphere passes a small circle. Through the corresponding points pqr on π pass 4 ellipses having double contact with γ and one of these is the required projection. The ambiguity arises from the 2-1 character of the correspondence and could be removed by selecting all the points PQR on one of the hemispheres bounded by T.

Since all great circles through a point P are concurrent at a diametrically opposite point Q, their projections will be a system of ellipses having double contact with γ and passing through two points p, q equidistant from the centre of γ. Fig. 1 represents such a system of ellipses.

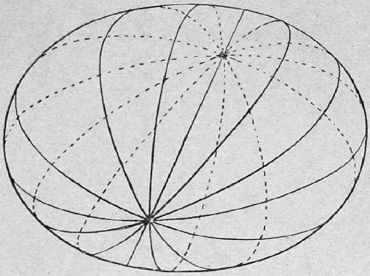

FIG. 1

Orthographic Projection

5. In every projection certain plane sections, namely those through O, will be represented by line ellipses. If these are not important sections, the disadvantage can be overlooked. The orthographic projection of the celestial sphere on a vertical plane has the advantage that γ is a circle but suffers from the serious defect that the horizon circle is represented by a line. Hence the East and West points overlap in the diagram while the path of any heavenly body due to diurnal motion is also a line ellipse. A parallel projection in a direction not parallel to the horizon circle, would appear to be ideal one for astronomical diagrams and is the one usually used.

How Spherical Triangles should be drawn

6. With the help of propositions I, II, III we may not only examine and declare a figure to be incorrect but also suggest how it should have been drawn. The points to examine are:

(i) Whether the curves representing great or small circles *touch* the bounding curve γ if they extend so far. ... (6·1)

(ii) In the case of great circles, whether the contact is at diametrically opposite points or would be if the ellipses were completed (assuming it is a case of parallel projection) See Fig. 1. ... (6·2)

Since a spherical triangle is formed of arcs of great circles, each of the sides must be an elliptic arc which *when completed* should touch γ at the extremities of a diameter. Fig. 2 shows such a triangle.

As to which way a side of the spherical triangle should curve, Prop. III gives an unambiguous answer which may be stated thus:—

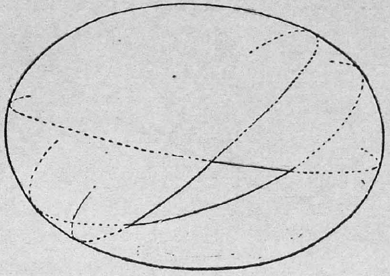

FIG. 2

Every great circle is represented by a curve which presents its concave side to the centre of the bounding ellipse γ. ... (6·3)

The observance of this rule would prevent a great many of the flagrant offences which are constantly perpetrated against the laws of perspective drawing.

7. Before concluding let me give illustrative examples of obviously wrong diagrams from well-known text-books on Spherical Trigonometry and Astronomy.

TODHUNTER and LEATHEM: *Spherical Trigonometry*, 1907.

Page 97, para 127: The great circles BFEC and ACDF should *touch* the bounding circle ABDE and should not have been drawn as if they had nodes at A, D, B and E. The Figure is wrong whether the projection is conical or parallel. (Prop. II.)

Page 165, para 206: All the lines in the figure are great circles and hence by (6·3) the centre of γ should be within the triangle ABC. The great circle $I_1 I_2$ should curve the other way so as to present its concave side to the centre.

Sir ROBERT BALL: *Spherical Astronomy*, 1923.

Page 383: para 127, Fig. 94: All the sides of the triangle RPN are concave to its interior so that by (6·3) the centre of γ is within the triangle. Hence R Ω should curve the other way so as to present its concave side.

SMART: *Text-book on Spherical Astronomy* 1931.

Page 15: Fig. 8. BC curves the wrong way. Similarly in Figs. 73 and 103 the curvatures of KX_1 and AS are wrong.

PAPERS ON HINDU ASTRONOMY—I
A METHOD OF COMPUTING THE BEGINNING OF THE INDIAN SOLAR YEAR

BY

DR. H. SUBRAMANI IYER, *Trivandrum.*

In all provinces of India various forms of ephemeris are computed to keep the public informed about the position of the Sun and the Moon and also to fix festivals and religious ceremonies depending upon the position of these and other celestial bodies. These methods of computation are interesting in that they are peculiar to India and have been handed down through centuries by those who studied the motion of these celestial bodies by continued naked eye observations. Computations are made even now according to the rules and aphorisms given in various Sanskrit Texts. It is possible that many of the texts now used in different places of India might either largely or to a small extent depend upon information contained in the *Suryasidhanta* and other ancient texts on Astronomy. One peculiar feature of these books is the very brief exposition of the intricate processes involved in accurately fixing the position of the Sun, Moon and planets. Large numbers are used in multiplication, division and other arithmetical processes for which there is no proper explanation. It is not assumed that the authors of the various texts did not know the rationale for the processes indicated in the brief stanzas given by them. They have not made it available to the generality of students who wanted to study astronomical calculations, and who are therefore blindly following the methods, without being able to give a rational explanation for them. This state of affairs is not peculiar to Astronomy alone; it is perhaps a common feature to all the sciences cultivated by the Ancient Indians. The Science of Ayurveda is another instance in point. I shall explain in the following pages the processes indicated in determining the Sun's longitude daily and also in fixing the *Sankranti*, or the beginning of a solar month. The rules given for fixing these are mentioned in a book called "Kriya Krama" which is followed in computing the yearly calendar in Kerala i.e., Travancore, Cochin and Malabar. In these places the era that is used is known as the Kollam era which began in 825 A.D. (In India various eras are used in various provinces.) In North Malabar the new year of this

era begins with the moment when the Sun enters Kanya Rasi whereas in other parts of Malabar, Cochin and Travancore the new year begins from the moment when the Sun enters Simha Rasi.

The Solar Year

One of the astronomical constants known to Indians from very early times is the length of the Solar or Sidereal year, which is the interval between two consecutive apparent passages of the Sun across a fixed point of the ecliptic. This value is mentioned in the *Surya Sidhanta* as 365·258756484 days and is slightly in excess of the Tropical year on which our present Western Civil Year is based. The length of the sidereal year as computed by a modern astronomer (W. M. Smart) is 365·2564 days, so that the Hindu year is in excess by 0·0023 of a day. The Hindu year is more nearly equal to the anomalistic year whose value according to modern astronomy is 365·2596 days, its difference from this being only 0·0008 of a day. It may very well be called "anomalistic" as it then easily leads us to calculate the Sun's anomaly.

Mesha Sankranti

Though there are many eras with different solar years starting from different positions of the sun in the ecliptic, the moment of the Sun entering the Mesha Rasi is an all-important event and marks the beginning of the new year in the well-known cycle of sixty years used in many parts of India, including the Tamil- and Telegu-speaking districts of Madras Province. Even in computing the ephemeris for other systems prevalent in other parts with different beginnings for the new year, the fixing of the moment *Mesha Sankranti* is important as from this the beginning of the other solar months can be easily calculated (Each solar month is the duration of the Sun's motion over each of the twelve equal divisions of the ecliptic beginning with the Mesha Rasi).

The rule followed by computers in Travancore, Cochin and other places to find the day of the week and exact time of *Mesha Sankranti* is as follows: [See Pancha Bodha or Kriya Krama for the original stanza, which with its endering in English is given here.]

शकाब्दशास्त्रार्थवधात्तिथीशेनासं दिनाद्यं कलिनाशपुण्यैः ।
गुर्वेशराद्यं सहितं सुनष्टं विवस्वतः सङ्क्रमणध्रुवं स्यात् ॥

1. Find the number of the year current at the time in Saka era (This era began in 78 A.D. and is used extensively in India and the

year in it begins with the moment of the Sun entering Mesha Rasi. This value is 747 *plus* the number of the year current in Kollam era which began in 825 A.D. as already mentioned.)

2. Multiply this number by 725 and divide the product by 576. Let the quotient be Q_1 and remainder R_1. The remainder given by $Q_1/7$ gives the week day of *Mesha Sankranti* counted from Sunday. Then, to get the exact moment of *Mesha Sankranti*, after sunrise of this day, assuming that the day is divided into 60 equal *nalis* or *ghatikas*, each *ghatik* into 60 equal *Vinalis* and each *Vinali* into 60 equal *Gurvaksharas*, R_1 is used as follows:

3. $R_1 \times 60/576 = K_1$ *nalis* plus $R_2/576$ say.

$R_2 \times 60/576 = K_2$ *Vinalis* plus $R_3/576$ say.

$R_3 \times 60/576 = K_3$ *Gurvaksharas* plus $R_4/576$ say and so on. Now add (K_1 *Nali* K_2 *Vina* K_3 *Gurva*) to 11 *Nali* 50 *Vina* 31 *Guruva* and we get the exact moment of *Mesha Sankranti* after the Sunrise of the day of the week already obtained. Perhaps the following example will make the process clearer.

Find the instant of *Mesha Sankranti* occurring in Kollam Year 1111 (corresponding to 1935-36 A.D.)

The Saka year corresponding to Kollam year 1111 is 1858 [being 1111 + 747]

Now $1858 \times 725/576 = 2338 + 362/576$. $R_1 = 362$

and $2338/7 = 334 + 0$.

Hence *Mesha Sankranti* of Kollam year 1111 occurred on a Sunday.

$362 \times 60/576 = 37$ *Nalis* plus $408/576$ ($K_1 = 37$, $R_2 = 408$)
$408 \times 60/576 = 42$ *Vina* plus $288/576$ ($K_2 = 42$, $R_3 = 288$)
$288 \times 60/576 = 31$ *Gurva* (nearly) ($K_3 = 31$)

	Na.	*Vin.*	*Gurva.*
Adding to	11	50	31
$(K_1 + K_2 + K_3)$ =	37	42	31
we get the exact time of *Mesha Sankranti* after Sunrise or Sunday to be	49	33	2

The above instant is for the standard meridian passing through Lanka. To get the corresponding time for any other place, say Trivandrum apply the correction (23 Vi. 40 Gu) and then we get the instant of *Mesha Sankramam* for Trivandrum.

The explanation of the working shown above is not evident and many Sanskrit scholars who were freely using this process were unable to explain why these operations are done and I therefore venture to give what seems to me to be a proper explanation of all the steps involved in the above process.

1. The Saka era should have begun with *Mesha Sankranti* on a Sunday at 11 *Na* 50 *Vi* 31 *Gu* after Sunrise. This mode of astronomical calculation should have been in existence in India, before 78 A.D. at least.

2. The length of a Solar year (being 365·25875 days) is 52 weeks and 1·25875 of a day. This latter quantity is equal to 725/576 of a day, up to 4 decimal places. Therefore with the advance of every Solar year, the Mesha Sankranti comes 1·2587 of a day later; and therefore after n Saka years it comes exactly $n \times 725/576$ days after a Sunday. From the quotient of this which is in days we can remove multiples of seven days and so this number is divided by 7 and the remainder used in counting days of the week from Sunday. As $R_1/576$ in $n \times 725/576$ gives the fraction of a day at which *Mesha Sankranti* takes place, this is converted into the lower denominations of *Nalis*, *Vinalis*, and *Gurvaksharams*.

GLEANINGS

66. It would be quite interesting to construct the geneological tree of each mathematical theory: in 90% of the cases, I am sure, one would find a race with an algebraic strain. Algebra is the Eve, though sometimes without Adam. The principal difficulty in the formation of the trees is that marriages between parents are numerous and cases of polygamy very frequent. Let me give some examples in telegraphic style:

(*a*) A Descriptive Geometry—*Descendants:* Differential geometry, projective Geometry.

(*b*) Projective Geometry—*Descendants of the 1st degree:* Theory of transformations and groups, Theory of invariants, Abstract Geometry (Möbius, Plücker, Staudt), *n*-dimensional geometry developed algebraically (Cayley, Klein, Veronese, Segre, etc.)

(*c*) Descendants of the 2nd degree—Non-Euclidean Geometry direct descendant of projective geometry and of differential geometry, Riemannian geometry, Analysis Situs.

(*d*) Descendants of the 3rd degree—(Through the marriage of groups, invariants, and functions) Groups continuous and discontinuous; automorphic functions.

(*e*) Descendants of the 4th degree—(Through the marriage between non-Euclidean Geometry, differential geometry and algebraic forms) Absolute differential calculus, general relativity, Levi-Civita parallelism.

(*f*) Descendants of the 5th degree—Spaces with affine and projective connections of Cartan, Schouten, Weyl, Veblen, etc.

Tr. from F. SEVERI'S Lecture: Le rôle de la géométric algébraique dans les mathématiques, Zurich Mathematical Congress.

LINES OF STRICTION ON A QUADRIC—

A SUPPLEMENTARY NOTE

BY

C. N. SRINIVASIENGAR, *Bangalore.*

1. A curve of *type* (m, n) on a quadric means a curve which meets any generator of one system in m points and any generator of the other system in n points. It is known that two curves of types (m, n) and (m', n') on a quadric intersect in $mn' + m'n$ points. The lines of striction of the two systems of generators are curves of types (1, 3) and (3, 1), and hence should meet in ten points. *Four of these are at infinity viz. the points where the section at infinity of the quadric touches the common tangents between the section and the circle at infinity.* This result was proved* by independent methods by Dr. B. Ramamurti and by the present writer. That the other six points are the six vertices of the surface is more or less a known result.† An algebraical method based on apolarity is given by Dr. Ramamurti (*loc. cit.*) The result can also be deduced from straightforward geometrical considerations, as follows :—

The central point of a given λ-generator is obtained thus :‡ Let P be the point at infinity on the generator, and q the section at infinity. Let R be the pole w.r.t. Ω (the imaginary circle at infinity of the tangent at P to q. If PR meets q again in T, the μ-generator through T meets the given generator at the central point of the latter.

Suppose now that this central point L is also the central point of the μ-generator through T. Then, evidently, the pole w.r.t. Ω, call it R′, of the tangent at T to q must lie along PT. Thus R and R′ both lie along PT. If V be the pole of PT w.r.t. q, it follows that the polars of V with respect to q and Ω are the same. It is easy to prove that a point will have the same polar with respect to two conics in general position when and only when the point is a vertex of their common self-polar triangle. If a side of this triangle cuts q in P and T, the λ-generator at P and the μ-generator at T intersect at a point which is the central point of either. Similarly the

* *Math. Student*, Vol. IV, 1936 p. 51.
Bull. Calcutta. Math. Soc., XXVIII, 1936 p. 49.

† Sommerville: *Anal. Geom. of Three Dimensions*, p. 305.

‡ H. F. Baker: *Principles of Geometry*, Vol. 3, p. 84.

μ-generator at P and the λ-generator at T have the same central point. Each side of the triangle thus leads to two intersections of the lines of striction*. That these points are the vertices of the quadric is easily verified. If ABC is the common self-polar triangle, and if O be the centre of the quadric, and a principal axis, say OB, meets the quadric in B_1, B_2, the tangent plane at B_1 or B_2 is parallel to the plane OAC. Hence the generators through B_1 or B_2 meet AC at the points where AC meets q.

We have thus obtained geometrically all the ten intersections of the two lines of striction.

2. *The case of the general paraboloid.—*

Let g_λ and g_μ be the generators at infinity, meeting at the point X. Let R_1 and R_2 be their poles w.r.t. Ω. The polar planes of R_1 and R_2 give two parabolas which constitute the two lines of striction. Using the previous arguments, it is easy to see that a λ-generator and a μ-generator have the same central point if and only if their points at infinity are collinear with R_1, R_2. In other words, if $R_1 R_2$ *i.e.* the polar of X w.r.t. Ω meets g_λ and g_μ in P and Q respectively, then the μ-generator through P and the λ-generator through Q have the same central point. This point and the point X are the only intersections of the two parabolas. Let V denote the vertex of the paraboloid so that VX is the axis. Since (X, P) and (X, Q) are conjugate points for Ω, VP and VQ are both perpendicular to VX. Hence VPQ is the tangent plane at the vertex. The generators at P and Q must therefore meet at the vertex.

This proves *geometrically* the result known from analytical considerations that each of the parabolas constituting a line of striction of the paraboloid passes through the vertex of the surface.

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* In this argument, P and T are supposed to be different points. If P and T coincide, then the line PRT becomes a common tangent to q and Ω. This is the case leading to the intersections at infinity for the lines of striction. See § 3 of my paper *loc. cit.*

ON THREE DESMIC TETRAHEDRA WHICH HAVE A COMMON ORTHOCENTRE AND ASSOCIATED SPHERES

BY

N. DURAIRAJAN, *Executive Engineer, Madras*

It is known that the three tetrahedra of a desmic system are self-polar with respect to a unique quadric. When the unique quadric is a sphere, the three tetrahedra are orthogonal (*i.e.* the opposite edges of each tetrahedron are perpendicular) and have a common othocentre at the centre of the polar sphere. The vertices of each of the tetrahedra are the centres of four spheres which are mutually orthogonal and orthogonal to the common polar sphere. The spheres of one tetrad of spheres cut the spheres of another tetrad at angles of $60°$ or $120°$. The above systems of spheres is partially analogous to the three tetrads of mutually orthogonal circles discussed by Dr. R. Vaidyanathasami. [R. 1].

While the tetrads of circles in the plane are mostly imaginary, the spheres in space have real centres and their radii in each tetrad are (i) all real, or (ii) three real and one imaginary. In the first case, the polar sphere is imaginary and in the second case, it is real.

2. Let $\Delta_1 = (A_0\ A_1\ A_2\ A_3)$, $\Delta_2 = (B_0\ B_1\ B_2\ B_3)$, $\Delta_3 = (C_0\ C_1\ C_2\ C_3)$ be the three tetrahedra. If Δ_1 is taken as the tetrahedron of reference, the vertices of Δ_2 and Δ_3 may be written $(1, \pm 1, \pm 1, \pm 1)$ and the unique self polar quadric is

$$x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0 \qquad \dots (1)$$

If Δ_1 is an orthogonal tetrahedron, the equation of the polar sphere can be written in the form (using tetrahedral co-ordinates).

$$P = x_0^2\alpha_0^2 + x_1^2\alpha_1^2 + x_2^2\alpha_2^2 + x_3^2\alpha_3^2 = 0 \qquad \dots (2)$$

where $A_rA_s^2 = \alpha_r^2 + \alpha_s^2$,
and $A_0\,A_1^2 + A_2A_3^2 = A_0\,A_2^2 + A_1A_3^2 = A_0\,A_3^2 + A_1A_2^2$.

$A_0\ A_1\ A_2\ A_3$ are the centres of spheres of radii $\alpha_0\ \alpha_1\ \alpha_2\ \alpha_3$ respectively which are mutually orthogonal: their common orthogonal sphere is P, whose centre is $H = (1/\alpha_0^2, 1/\alpha_1^2, 1/\alpha_2^2, 1/\alpha_3^2)$ and radius is $\omega \cdot \sqrt{(-1)}$, where

$$\frac{1}{\omega^2} = \frac{1}{\alpha_0^2} + \frac{1}{\alpha_1^2} + \frac{1}{\alpha_2^2} + \frac{1}{\alpha_3^2} \qquad \dots$$

The sphere P is imaginary if the spheres (A_0, α_0) (A_1, α_1) $(A_2\ \alpha_2)$ $(A_3\ \alpha_3)$ are all real.

The vertices of Δ_2 and Δ_3 are

$$\left(-\frac{1}{\alpha_0}, \frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \frac{1}{\alpha_3}\right), \left(\frac{1}{\alpha_0}, \frac{-1}{\alpha_1}, \frac{1}{\alpha_2}, \frac{1}{\alpha_3}\right), \left(\frac{1}{\alpha_0}, \frac{1}{\alpha_1}, \frac{-1}{\alpha_2}, \frac{1}{\alpha_3}\right),$$

$$\left(\frac{1}{\alpha_0}\ \frac{1}{\alpha_1}\ \frac{1}{\alpha_2}\ \frac{-1}{\alpha_3}\right) \text{ and } \left(\frac{1}{\alpha_0}, \frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \frac{1}{\alpha_3}\right), \left(\frac{1}{\alpha_0}\ \frac{1}{\alpha_1}\ \frac{-1}{\alpha_2}\ \frac{-1}{\alpha_3}\right),$$

$$\left(\frac{1}{\alpha_0}\ \frac{-1}{\alpha_1}\ \frac{1}{\alpha_2}\ \frac{-1}{\alpha_3}\right), \left(\frac{1}{\alpha_0}\ \frac{-1}{\alpha_1}\ \frac{-1}{\alpha_2}\ \frac{1}{\alpha_3}\right).$$

The tetrahedra $\Delta_2\ \Delta_3$ have P as self-polar sphere and have H as common orthocentre. The equations of the faces of the tetrahedra are

for Δ_2, $$Y_0 = -x_0\alpha_0 + x_1\alpha_1 + x_2\alpha_2 + x_3\alpha_3 \qquad \dots (3)$$

for Δ_3 $$Z_0 = x_0\alpha_0 + x_1\alpha_1 + x_2\alpha_2 + x_3\alpha_3 \qquad \dots (4)$$

It is easily seen that

$$\tfrac{1}{4}\ \Sigma Y_0^2 = \tfrac{1}{4}\Sigma Z_o^2 = x_0^2\alpha_0^2 + x_1^2\alpha_1^2 + x_2^2\alpha_2^2 + x_3^2\alpha_3^2 \qquad \dots (5)$$

3. *The radii of the spheres associated with Δ_2 and Δ_3*

Let $(\beta_0\ \beta_1\ \beta_2\ \beta_3)$ and $(\gamma_0\ \gamma_1\ \gamma_2\ \gamma_3)$ be the radii of the two tetrads of spheres.

$$\frac{\text{Vol. } HB_1\ B_2\ B_3}{\text{Vol. } B_0\ B_1\ B_2\ B_3} = \frac{\omega^2}{\beta_0^2} \qquad \dots (6)$$

We get relations between the radii

$$\left.\begin{aligned} \frac{2}{\beta_0} &= \frac{-1}{\alpha_0} + \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3} \\ \frac{2}{\beta_1} &= \frac{1}{\alpha_0} - \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3} \\ \frac{2}{\beta_2} &= \frac{1}{\alpha_0} + \frac{1}{\alpha_1} - \frac{1}{\alpha_2} + \frac{1}{\alpha_3} \\ \frac{2}{\beta_3} &= \frac{1}{\alpha_0} + \frac{1}{\alpha_1} + \frac{1}{\alpha_2} - \frac{1}{\alpha_3} \end{aligned}\right\} \qquad \dots (7)$$

$$\left.\begin{aligned} \frac{2}{\gamma_0} &= \frac{1}{\alpha_0} + \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3} \\ \frac{2}{\gamma_1} &= \frac{1}{\alpha_0} + \frac{1}{\alpha_1} - \frac{1}{\alpha_2} - \frac{1}{\alpha_3} \\ \frac{2}{\gamma_2} &= \frac{1}{\alpha_0} - \frac{1}{\alpha_1} + \frac{1}{\alpha_2} - \frac{1}{\alpha_3} \\ \frac{2}{\gamma_3} &= \frac{1}{\alpha_0} - \frac{1}{\alpha_1} - \frac{1}{\alpha_2} + \frac{1}{\alpha_3} \end{aligned}\right\} \qquad \dots (8)$$

4. A sphere of one tetrad is isogonal to the spheres of the other two tetrads.

We make use of the following theorem [R. 2]. "The isogonal (*i.e.* cutting at equal angles) spheres of four given spheres form eight coaxal pencils of spheres. The radical plane of each pencil is a plane of similitude of the four spheres and the common orthogonal sphere of the four given spheres belongs to the eight pencils." If the spheres be $(A_0\, \alpha_0)$ $(A_1\, \alpha_1)$ $(A_2\, \alpha_2)$ $(A_3\, \alpha_3)$, then the eight planes of similitude are the eight planes $Y_0\, Y_1\, Y_2\, Y_3$ and $Z_0\, Z_1\, Z_2\, Z_3$ and the tetrads of spheres of radii $\beta_0\, \beta_1\, \beta_2\, \beta_3$ and $\gamma_0\, \gamma_1\, \gamma_2\, \gamma_3$ are isogonal to the fundamental tetrad of spheres. It can be proved that the angles of intersection of the spheres are either 60° or 120°. The sphere $(B_0\, \beta_0)$ cuts $(A_1\, \alpha_1)$ $(A_2\, \alpha_2)$ $(A_3\, \alpha_3)$ at 60° and $(A_0\, \alpha_0)$ at 120°.

5. The common orthocentre H lies on the radical axis of the circumspheres of $\triangle_1\ \triangle_2\ \triangle_3$.

This follows from the relation [R. 3]

$$R^2 - OH^2 = 3\omega^2 \qquad \ldots (9)$$

where R is the circumradius of the tetrahedron and O is the circumcentre.

References.

[R. 1]. R. Vaidyanathaswami "On systems of four mutually orthogonal circles." *Cambridge Philosophical Society.* Vol. XXIV. Part IV. ... P. 578.

[R. 2] N. Altschiller Court. *Modern Pure Solid Geometry.* ... P. 224.

[R. 3] ... ... P. 271.

GLEANINGS

Months—for sale

67. It is quite characteristic of these people, (the Romans) great only in war and as citizens of the state, despising all more refined civilization, that they were satisfied for over 700 years with this highly confused reckoning of the year and turned over to the discretion of their priests the interpolation of the months. We know from history that the latter often prostituted their interpolated months for money; magistrates who wanted to remain somewhat longer in office, had a month inserted by them if the new candidates did not offer the opposite by greater credit or more money. Julius Caesar first made an end of the nuisance, but with foreign aid: an Egyptian, Sosigenes, had to give him an improved calendar which we still know by his name as Julian.

GAUSS: *Inaugural Lecture on Astronomy,* Gottingen (1808 ?)

A FEW RESULTS IN TRIANGLE-GEOMETRY

BY

N. DURAIRAJAN

1. This note deals with a few results in the geometry of the Triangle obtained by the use of Complex numbers. ABC is the fundamental triangle and the complex co-ordinates of its vertices are α, β and γ. The notation A $(=\alpha)$ means that the complex co-ordinate of A is $\alpha = x_1 + i\, y_1$.

2. *The Beltrami points.* The Hessian or isodynamic points, $H_+(=h_+)$ and $H_-(=h_-)$ are expressible in the form

$$h_- = -\left(\frac{\beta\gamma + \omega\gamma\alpha + \omega^2\alpha\beta}{\alpha + \omega\beta + \omega^2\gamma}\right) \qquad \dots \quad (1)$$

$$h_+ = -\left(\frac{\beta\gamma + \omega^2\gamma\alpha + \omega\alpha\beta}{\alpha + \omega^2\beta + \omega\gamma}\right) \qquad \dots \quad (2)$$

where ω is an imaginary cube root of unity.

Let $M_1\,(=\mu_1)$ and $M_2\,(=\mu_2)$ be points given by

$$\mu_1 = \frac{\beta^2\gamma + \gamma^2\alpha + \alpha^2\beta - 3\alpha\beta\gamma}{\alpha^2 + \beta^2 + \gamma^2 - \beta\gamma - \gamma\alpha - \alpha\beta} \qquad \dots \quad (3)$$

$$\mu_2 = \frac{\beta\gamma^2 + \gamma\alpha^2 + \alpha\beta^2 - 3\alpha\beta\gamma}{\alpha^2 + \beta^2 + \gamma^2 - \beta\gamma - \gamma\alpha - \alpha\beta} \qquad \dots \quad (4)$$

Then we have the obvious relations

$$h_- + \omega^2\mu_1 + \omega\mu_2 = 0 \qquad \dots \quad (5)$$

$$h_+ + \omega\mu_1 + \omega^2\,\mu_2 = 0 \qquad \dots \quad (6)$$

$$\therefore \quad h_- + h_+ = \mu_1 + \mu_2 \qquad \dots \quad (7)$$

Hence H_+ and H_- are the vertices of the two equilateral triangles described on $M_1\,M_2$ as base. The points $M_1\,M_2$ are the Beltrami points* and the line joining them is the Beltrami line, (usually called the Lemoine line or axis).

(ii) We have

$$\mu_1 - \alpha = \frac{(\gamma - \alpha)(\alpha - \beta)^2}{\theta^2} \qquad \dots \quad (8)$$

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* The Beltrami points are the inverses of the Brocard points with respect to the circumcircle. Please see Morley and Morley "*Inversive Geometry*" P. 78. The properties of the Hessian points are discussed by Frank Morley in his paper on the "Covariant Geometry of the Triangle" *Quarterly Journal of Mathematics.* Vol. 25. P. 186.

$$\mu_1 - \beta = \frac{(\alpha - \beta)(\beta - \gamma)^2}{\theta^2} \qquad \ldots \quad (9)$$

$$\mu_1 - \gamma = \frac{(\beta - \gamma)(\gamma - \alpha)^2}{\theta^2} \qquad \ldots \quad (10)$$

$$\mu_1 - \mu_2 = \frac{-(\beta - \gamma)(\gamma - \alpha)(\alpha - \beta)}{\theta^2} \qquad \ldots \quad (11)$$

where $$\theta^2 = \alpha^2 + \beta^2 + \gamma^2 - \beta\gamma - \gamma\alpha - \alpha\beta \qquad \ldots \quad (12)$$

Equating moduli, we deduce that the tripolar co-ordinates of M_1 M_2 are $(bc^2 : ca^2 : ab^2)$ and $(b^2c : c^2a : a^2b)$.

The Beltrami points lie outside the cirumcircle for, $OH_+ . OH_- = R^2$

$$\text{and } \left(\frac{OH_+ + OH_-}{2}\right)^2 = R^2 + \left(\frac{OH_+ - OH_-}{2}\right)^2$$

so that the mid-point of H_+ H_- (which is also the middle point of M_1 M_2) lies outside the circumcircle and hence the Beltrami line is outside the circumcircle. Knowing that the Brocard points lie within the circumcircle, that their tripolar co-ordinates, are $(bc^2 : ca^2 : ab^2)$ and $(b^2c : c^2a : a^2b)$ and that two inverse points have the same tripolar co-ordinates, we deduce that the Beltrami points are the inverses of the Brocard points with respect to the circumcircle.

(iii) Equating arguments, we get

$$A\hat{M}_1B = B - C, \quad B\hat{M}_1C = C - A, \quad C\hat{M}_1A = A - B,$$

neglecting multiples of π. We get similar relation for M_2.

(iv) As $M_1A \cdot a : M_1B \cdot b : M_1C \cdot c = c : a : b$, the pedal triangles of M_1 M, are similar to the fundamental triangle.

(v) As $\mu_1 - \alpha : \alpha - \mu_2 : \mu_2 - \mu_1 = \alpha - \beta : \alpha - \gamma : \beta - \gamma$ the triangles AM_1M_2, BM_1M_2, CM_1M_2 are inversely similar to the triangle ABC.

(vi) $AM_1 \cdot BM_1 \cdot CM_1 = AM_2 \cdot BM_2 \cdot CM_2$

3. *The F-points of a Triangle.*

The points F_1 $(=\phi_1)$ F_2 $(=\phi_2)$ F_3 $(=\phi_3)$ are given by

$$\phi_1 = \frac{\alpha(\beta - \gamma)^2 + \beta(\gamma - \alpha)^2 + \gamma(\alpha - \beta)^2}{2\theta^2} \qquad \ldots \quad (13)$$

$$\phi_2 = \frac{\beta(\beta - \gamma)^2 + \gamma(\gamma - \alpha)^2 + \alpha(\alpha - \beta)^2}{2\theta^2} \qquad \ldots \quad (14)$$

$$\phi_2 = \frac{\gamma(\beta - \gamma)^2 + \alpha(\gamma - \alpha)^2 + \beta(\alpha - \beta)^2}{2\theta^2} \qquad \ldots \quad (15)$$

$$\therefore \quad \phi_1 + \phi_2 + \phi_3 = \alpha + \beta + \gamma \qquad \ldots (16)$$

$$2\phi_1 = \mu_1 + \mu_2 \qquad \ldots (17)$$

$$\frac{\phi_2 + \mu_1}{2} = \frac{\alpha + \beta + \gamma + \mu_2}{4} \qquad \ldots (18)$$

$$\phi_2 - \phi_3 = -\frac{3}{2}(\mu_1 - \mu_2) \qquad \ldots (19)$$

Hence F_1 is the mid-point of $M_1 M_2$ and $F_2 F_3$ is parallel to $M_1 M_2$

(ii) If $f(z) = (z-\alpha)(z-\beta)(z-\gamma)$, then the roots of $f'(z) = 0$ are the foci of the maximum inscribed ellipse. If the foci are $S_1(=\sigma_1)$ and $S_2(=\sigma_2)$, then

$$3(z-\sigma_1)(z-\sigma_2) = 3z^2 - 2z(\alpha+\beta+\gamma) + \beta\gamma + \gamma\alpha + \alpha\beta \qquad \ldots (20)$$

If $\therefore$ $G(=g)$ be the centroid, $g = \frac{1}{3}(\alpha+\beta+\gamma)$

$$\sigma_1 + \sigma_2 = \tfrac{2}{3}(\alpha+\beta+\gamma) = 2g \qquad \ldots (21)$$

$$\sigma_1 \sigma_2 = \tfrac{1}{3}(\beta\gamma + \gamma\alpha + \alpha\beta) \qquad \ldots (22)$$

$$\sigma_1 = g + \theta/3 \qquad \ldots (23)$$

$$\sigma_2 = g - \theta/3 \qquad \ldots (24)$$

$$(\phi_2-\phi_3)^2 + (\phi_3-\phi_1)^2 + (\phi_1-\phi_2)^2 = \tfrac{1}{4}\Sigma(\beta-\gamma)^2 \qquad \ldots (25)$$

$$= \Sigma\left(\frac{\alpha+\beta}{2} - \frac{\alpha+\gamma}{2}\right)^2$$

$\therefore$ The maximum inscribed ellipse of the triangle $F_1 F_2 F_3$ is confocal with the maximum inscribed ellipse of the medial triangle of ABC.

(iii) $$\alpha - \phi_1 = \frac{3(\alpha-\beta)(\alpha-\gamma)(\alpha-g)}{2\theta^2} \qquad \ldots (26)$$

$$\beta - \phi_1 = \frac{3(\beta-\gamma)(\beta-\alpha)(\beta-g)}{2\theta^2} \qquad \ldots (27)$$

$$\gamma - \phi_1 = \frac{3(\gamma-\alpha)(\gamma-\beta)(\gamma-g)}{2\theta^2} \qquad \ldots (28)$$

$$\therefore \quad F_1A : F_1B : F_1C = \frac{AG}{BC} : \frac{BG}{CA} : \frac{CG}{AB} \qquad \ldots (29)$$

Using the result in my former paper* it can be deduced that F_1 is the focus of the biparabola which touches the six joins of the four points A, B, C, G. Also, the pedal triangle GF_1 with respect to $\triangle$ ABC has its sides proportional to a triangle formed by the three medians and each side subtends at F_1 an angle equal to the algebraic sum of the angles which it subtends at G and the opposite vertex. F_1 is the inverse of the Symmedian point with respect to the circumcircle.

(iv) A curve of class three which touches the mutual joins of two triads of points A, B, C, and DEF is

$$abc = k \cdot def \qquad (k = \text{const.})$$

* *Jour. Ind. Math. Soc.* Vol. 17. P. 224.

where a, b, c, d, e, f are the tangential equations of the six points. When D, E, F lie on the sides BC, CA, AB of $\triangle$ABC and divide them in equal ratios, the curve, referred to $\triangle$ABC and using areal tangential co-ordinates, is

$$lmn = k\left(\frac{m+n\lambda}{1+\lambda}\right)\left(\frac{n+l\lambda}{1+\lambda}\right)\left(\frac{l+m\lambda}{1+\lambda}\right) \quad \ldots (30)$$

When $k=1$, the curve is a biparabola *i.e.* has the line at infinity as a bitangent. Reducing, we get

$$\lambda(mn^2+nl^2+lm^2-3lmn)+(m^2n+n^2l+l^2m-3lmn)=0$$

The focus of the curve is obtained by writing

$l=z-\alpha,\ m=z-\beta,\ n=z-\gamma$ and is given by

$$z=\frac{\mu_1+\mu_2\lambda}{1+\lambda} \quad \ldots (31)$$

$\therefore$ The foci of the pencil of biparabolas lie on the line $\mu_1\mu_2$ and divides that segment in the same ratio. μ_1 and μ_2 are the foci of the biparabolas $m^2n+n^2l+l^2m-3lm=0$ and $mn^2+nl^2+lm^2-3lmn=0$ and the $\triangle$ABC is doubly incident on the curves *i.e.* the vertices are on the curve and the sides are tangents.

When $\lambda=1$, the biparabola

$$l(m-n)^2+m(n-l)^2+n(l-m)^2=0 \quad \ldots (32)$$

touches the sides and the medians of the triangle. The focus is the mid-point of the segment joining the Hessian points. This biparabola is the envelope* of the asymptotes of the conics through the four points A, B, C, G.

4. *Artzt's Parabolas* *

Artzt's first parabola touches the sides AB, AC at B and C respectively. Its areal tangential equation is

$$l^2-mn=0 \quad \ldots (33)$$

Its focus is given by

$$(z-\alpha)^2=(z-\beta)(z-\gamma)=0$$

$$\therefore \quad z=(\beta\gamma-\alpha^2)\,/\,(\beta+\gamma-2\alpha) \quad \ldots (34)$$

The focus is obviously the mid-point of the line joining $A(=\alpha)$ and the corresponding Counter point, $A'(=\alpha')$ whose co-ordinate is given by

$$\frac{2\beta\gamma-\gamma\alpha-\alpha\beta}{\beta+\gamma-2\alpha}=\alpha' \quad \ldots (35)$$

The foci of the three Artzt's parabolas are the vertices of Brocard's second triangle. The lines AA′, BB′, CC′ are concurrent at the symmedian point.

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* See *Jour. Ind. Math. Soc.* Vol. XVIII (Part II) P. 108.

* Please see Gallatly's "*The Modern Geometry of the Triangle*" 2nd Ed. P. 88.

CORRELATION IN MINGLED RECORDS

BY

S. SUBRAMANIAN, *Annamalainagar*.

1. Two variates may be uncorrelated in each of several records separately, but when these records are mingled, a correlation generally results. This fact was pointed out by Karl Pearson many years ago; Prof. Yule has discussed the case of two records in his book on the *Theory of Statistics*. But the more general problem of studying the correlation that results from mingling a number of records *each one of which shows a correlation* seems to have escaped attention. The present note is an attempt at solving this problem.

2. Let N_1; N_2; . . . denote the number of observations in the various records and x, y the variates in question;

let

M_{1x}, M_{1y}; M_{2x}, M_{2y}; . . . . . . be the means

σ_{1x}, σ_{1y}; σ_{2x}, σ_{2y}: . . . . . . the standard deviations

r_1; r_2; . . . . . . the correlation coefficients,

and further let $\overline{M}_x$, $\overline{M}_y$; $\overline{\sigma}_x$, $\overline{\sigma}$; ρ stand for the corresponding quantities after mingling.

The product-sum of the deviations with respect to the general mean (after mingling) is

$$\sum N_1 r_1 \sigma_{1x} \sigma_{1y} + \sum N_1 (\overline{M}_x - M_{1x})(\overline{M}_y - M_{1y}).$$

Now $\sum N_1 (\overline{M}_x - M_{1x})(\overline{M}_y - M_{1y})$

$$= \overline{M}_x \sum N_1 (\overline{M}_y - M_{1y}) - \overline{M}_y \sum N_1 M_{1x} + \sum N_1 M_{1x} M_{1y}.$$

Since $\overline{M}_x \sum N_1 = \sum N_1 M_{1x}$ and $\overline{M}_y \sum N_1 = \sum N_1 M_{1y}$,
the above expression reduces to

$$\{\sum N_1 \cdot \sum N_1 M_{1x} M_{1y} - \sum N_1 M_{1x} \cdot \sum N_1 M_{1y}\} / \sum N_1 \qquad \ldots \quad (1)$$

$$= \sum N_1 N_2 (M_{1x} - M_{2x})(M_{1y} - M_{2y}) / \sum N_1$$

the number of terms in the numerator being equal to the number of ways of combining the records two at a time.

The product sum of the deviations now becomes

$$\sum N_1 r_1 \sigma_{1x} \sigma_{1y} + \sum N_1 N_2 (M_{1x} - M_{2x})(M_{1y} - M_{2y}) / \sum N_1.$$

The values $\overline{\sigma_x}$ and $\overline{\sigma_y}$ can be deduced from formula (I) by replacing the r's by unity and y by x or x by y as the case may be.

Thus

$$\rho = \frac{\sum N_1 r_1 \sigma_{1x} \sigma_{1y} + \sum N_1 N_2 (M_{1x} - M_{2x})(M_{1y} - M_{2y}) / \sum N_1}{\left[\sum N_1 \sigma_{1x}^2 + \frac{\sum N_1 N_2 (M_{1x} - M_{2x})^2}{\sum N_1}\right]^{1/2} \left[\sum N_1 \sigma_{1y}^2 + \frac{\sum N_1 N_2 (M_{1y} - M_{2y})^2}{\sum N_1}\right]^{1/2}} \quad \ldots \ldots \text{(II)}$$

3. (*a*) It may be observed at once that ρ will not necessarily be positive even if all the r's are positive. Much depends on the nature of the values of M_{1x}, M_{1y} : M_{2x}, M_{2y} ; etc. Thus cases may arise where *two characteristics are positively correlated in each record separately, but when the records are mingled the two become negatively correlated.* Mingling may also turn a negative correlation in the records into a positive one.

(*b*) Let us suppose that $r_1 = r_2 = \cdot \cdot \cdot \cdot = 0$; then the numerator in the expression for ρ becomes dependent on

$$\sum N_1 N_2 (M_{1x} - M_{2x})(M_{1y} - M_{2y}).$$

It is clear that ρ will be zero if every one of these terms vanishes ; that is, if every pair of records has at least one mean in common ($\bar{x}$ or $\bar{y}$). It should be noticed that we have here a condition which is sufficient but not necessary if there are more than two records. If there are only two records there is only one term in the $\sum$ and it can be seen that ρ becomes zero only if either $M_{1x} = M_{2x}$ or $M_{1y} = M_{2y}$.

Many more particular cases may be considered individually, but it appears sufficient here to mention that formula II of section (2) will furnish a starting point for the discussion of all such cases.

On Squares in Arithmetical Progression

1. Dr. Alfred Moessner * has asked for a general solution to the equations:

$$x^2 - y^2 = y^2 - z^2 = a^{2n-1}.$$

As an example, he gives $21^2 - 15^2 = 15^2 - 3^2 = 6^3$.

The equations appear to be interesting and a general solution may be worthy of record. Dr. Moessner is in possession of a general solution, I believe, but his solution is not known to me.

2. First consider the equation:

(A) $x^2 - y^2 = y^2 - z^2$, or $(x+y)(x-y) = (y+z)(y-z)$

Since $x^2 + z^2 = 2y^2$, we must have $x \equiv y \equiv z$ (mod. 2).

Let $x + y = 2u_1$, $x - y = 2u_2$; $y + z = 2v_1$, $y - z = 2v_2$; then (A) takes the form †:

(B) $u_1 u_2 = v_1 v_2$, where $u_1 - u_2 = v_1 + v_2$.

The most general solution to (B) is given by

$$u_1 = m_1 m_2,\ u_2 = m_3 m_4;\ v_1 = m_1 m_3,\ v_2 = m_2 m_4;$$

where $(m_2, m_3) = 1$.

Since $u_1 - u_2 = v_1 + v_2$, we must have $m_1/m_4 = (m_2 + m_3)/(m_2 - m_3)$.

Let $g = (m_2 + m_3,\ m_2 - m_3) = 1$ or 2.

Then $u_1 = t\, m_2 (m_2 + m_3)/g$, $u_2 = t\, m_3 (m_2 - m_3)/g$;

$v_1 = t\, m_3 (m_2 + m_3)/g$, $v_2 = t\, m_2 (m_2 - m_3)/g$.

Hence $x = t' (m^2{}_2 + 2\, m_2 m_3 - m^2{}_3)$, $y = t' (m^2{}_2 + m^2{}_3)$,

and $z = t' (m^2{}_3 + 2\, m_2 m_3 - m^2{}_2)$; where $t' = t/g$.

Now consider the equation: $4u_1 u_2 = a^{2n-1}$;

let $4\, m_2 m_3 (m^2{}_2 - m^2{}_3)/g^2 = h_1 h^3{}_2 h^5{}_3 \ldots h_n{}^{2n-1}$.

Then $t = h_1{}^{n-1} h_2{}^{n-2} h_3{}^{n-3} \ldots h_{n-1} \cdot k^{2n-1}$, and $a = h_1 h_2 h_3 \ldots h_n \cdot k^2$.

As another example of Moessner's type, we have

$$690^2 - 510^2 = 510^2 - 210^2 = 60^3.$$

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* In his letter of the 7th November, 1937.

† Such equations are considered in my paper on "A Problem in Diophantine Analysis"—*American Jour. of Math.*, 56, (1934), 269—274.

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On Descarte's Rule of Signs

Descarte's Rule of Signs asserts that an equation $f(x)=0$ cannot have more positive roots than there are changes of sign in the terms of the polynomial $f(x)$. In this note I discuss the situation in greater detail than most text-books several of which merely *verify* the result.

We have to show that if a polynomial $f(x)$ be multiplied by $x-a$, $a>0$, there will be at least one more change of sign. We say that "a change occurs in the rth place" if that place differs in sign from that of the $(r-1)^{th}$ place.

The resulting polynomial P is obtained by summing two expressions A and B, A having the signs of the original polynomial in order and B having all these signs reversed. Also B begins at one place to the right of A, so that any place in A will have underneath it a sign opposite to the preceding place of A.

Let us consider the signs of the terms in A, B and P when any term is absent in the original polynomial we shall write a zero in its place. In counting the changes of sign, the zeros are not taken into account.

Let a change occur in the lth place in the original polynomial and the next change in the mth place, so that all the intervening places have either zeros in them or are of the same sign as the lth place. Hence the lth and m^{th} places have opposite signs. Since a change occurs in the lth place in A, the preceding place of A has either an opposite sign or a zero in it. The lth place of A has therefore underneath it the same sign or a zero and in either case the lth place of P has the same sign as the lth place of A. In general, if a change occurs in any place in A, in that place, the sign of P is the same as that of A. The mth place in P has, similarly, the same sign as the mth place of A so that the signs of the lth and m^{th} places of P differ. Hence whatever signs there be in P between these two places a change of sign occurs either between these two places or at the second of the two. If the signs in question are all the same as that of the lth place, there is a change of sign in the mth place. If the signs are all the same as that of the mth place, there is a change of sign in the $(l+1)^{th}$ place. If these signs are both positive and negative there are more changes of sign. Thus whenever two places in P differ in sign, there must be at least one change of sign either between them or at the second of the places,

whatever be the intervening signs. This result is not altered even if some or all of the intervening places have zeros.

We see, therefore, that when a change occurs in the mth place (after the lth place) in A, at least one change occurs in P, not later than the mth place. Further this occurrence of change in P is after the lth place and hence corresponds to the change of the mth place of P. Thus corresponding to every change in A, there occurs at least one change in P either in the same place or before that place. In particular, if the change in the lth place is the first change, the first and the lth places of A and therefore of P are of opposite signs and hence a change of sign occurs in P not later than the lth place.

We have now to prove that there is at least one change in P in addition to the changes that occur in it corresponding to all the changes of A.

Let the last change in A occur in the nth place. Then as in the other cases where a change occurs, the nth place of P has the same sign as the nth place of A. Also the last sign of P is the same as the last sign of B, that is a sign opposite to the last sign of A which is the same as that of the nth place since there is no change in A after this place. Hence the nth and last places of P differ in sign. Therefore, there is at least one change of sign after the nth place in it. This result is unaltered even if some zeros occur about these places either in A or B or P. This change in P is an additional one since any of the changes of P that corresponds to a change in A occurs at the most in the nth place but not later than that.

Hence the effect of multiplying a polynomial by a binomial corresponding to a positive root is to introduce at least one more change of sign. The rest of the proof easily follows.

Masulipatam.

M. P. RAMA SASTRY,
Lecturer, Nobel College.

ASTRONOMICAL NOTES

For February, March and April 1938

Sun. During the period from 1st February to the end of April the Sun will be moving in an easterly direction from 20 h. 55 m. R.A. to 2 h. 29 m. R.A. and its Declination will be decreasing from 17° 24′ S. to 0° 0′ on March 21st and then it increases to 14° 45′ North by the end of April. On 21st March the Sun is at the Vernal equinox and hence the duration of day and night will be the same. The Sun's path for this period will start from about the centre of Capricornus and passing Aquarius, Pisces will end in the middle of Aries by the end of April.

The Sun's position at the beginning of each month will be as follows for Trivandrum :

	1st February	*1st March*	*1st April*
R. A. ...	20 h 55 m	22 h 44 m	0 h 38 m
Decl. ...	17 ° 24′S	8 ° 0′S	4 ° 7′ N.

Moon. The following phases of the Moon will occur during the months of February, March and April :

	February I. S. T.	*March* I. S. T.	*April* I. S. T.
New Moon	—	2nd at 11-10 A.M.	1st at 0-22 A.M. 30th at 10-58 A.M.
First Quarter	8th at 6- 3 A.M.	9th at 2-5 P.M.	7th at 8-40 P.M.
Full Moon	14th at 10-44 P.M.	16th at 10-45 A.M.	14th at 11-51 P.M.
Last Quarter	22nd at 9-54 A.M.	24th at 6-36 A.M.	23rd at 1-44 A.M.

The Moon will be nearest the earth (perigee) on 12th February, 11th March and 5th April and will be farthest from the earth (apogee) on 24th February, 23rd March and 20th April.

The moon's position at the beginning of each month will be as follows :—

	February	*March*	*April*
R. A. ...	21 h. 11 m.	21 h 44 m.	0 h 45.
Decln. ...	11° 13′ S.	8° 21′ S.	8° 42′ N.

Mercury. During the early part of February this planet is a morning object and may be seen near the eastern horizon just before

Sunrise. By the end of the month it will be too near the Sun to be visible and it appears as an evening object from the middle of March to the middle of April and may be seen near the western horizon just after Sunset. Mercury will be in conjunction with Jupiter on 17th February and with Venus on 20th March and 8th April. The planet will be in superior conjunction with the Sun on 8th March and in inferior conjunction with it on the 21st April. On 2nd April the planet is at its greatest elongation 19° East and on the same day it is in conjunction with the Moon. After 11th April the date of its stationary position the motion of the planet is retrograde. The path of the planet is from Sagittarius to Aries during this period. The position of the planet is defined as follows during the beginning of each month.

	February	*March*	*April*
R. A. ...	19 h 23 m.	22 h 23 m.	1 h 45 m.
Decln. ...	22° 25′ S	12° 23′ S	13° 19′ N.

Venus. During the month of February Venus will be setting with the Sun and from March the planet will be visible as an evening object seen near the western horizon just after Sunset. Venus will be in superior conjunction with the Sun on 4th February and will be in conjunction with Saturn, Moon and Uranus on 18th March, 1st April and 15th April respectively. The path of the planet is from Capricornus to Taurus during this period.

The position of the planet is as follows:—

	February	*March*	*April*
R. A. ...	20 h 53 m.	23 h 9 m.	1 h 30 m.
Decln. ...	18° 44′ S	7° 2′ S	8° 36′ N.

Mars. During February Mars will be in Pisces and passing Aries on the 12th April enters Taurus. Mars is an evening object during this period and will be seen on the western sky. This planet is in conjunction with Saturn on 2nd February and with Uranus on the 28th March. It will be in conjunction with the Moon on the following dates 4th February, 5th March and 3rd April.

The position of the planet will be as follows:

	February	*March*	*April*
R. A. ...	0 h 4 m.	1 h 20 m.	2 h 45 m.
Decln. ...	0° 0′ N.	8° 25′ N.	16° 21′ N.

Jupiter. Like Venus this planet will not be visible during the early part of February being lost in the rays of the Sun. From March

onward Jupiter will be visible as a morning object rising earlier day by day. The planet will be in Capricornus till 29th March and then it enters Aquarius and remains in it for the rest of the period. Jupiter will be in conjunction with the moon on the following dates: 28th February, 28th March and 25th April.

Its positions will be as follows :—

	February	*March*	*April*
R. A. ...	20 h 50 m.	21 h 16 m.	21 h 43 m.
Decln. ...	18° 19′ S.	16° 29′ S.	14° 28′ S.

Saturn. Saturn will be in Pisces throughout the period. The planet is an interesting evening object and its rings are gradually widening. By the end of March it will be nearing the Sun and on the 29th March it will be in conjunction with it. By the middle of April the planet will be visible as a morning object. It will be in conjunction with the moon on the following dates: 4th February, 4th March and 28th April.

The position of the planet will be as follows;

	February	*March*	*April*
R. A. ...	0 h 9 m.	0 h 20 m.	0 h 34 m.
Decln. ...	1° 28′ S.	0′ 14′ S.	1° 17′ N.

Uranus. The planet will be visible during the evening hours through large telescopes.

The position of the planet is as follows :—

	February	*March*	*April*
R. A. ...	2 h 30 m.	2 h 33 m.	2 h 38 m.
Decln. ...	14° 22′ N.	14° 36′ N.	15° 2′ N.

Neptune. This planet will be visible throughout the night. It will be in opposition with the Sun on 11th March.

The position of the planet will be as follows :—

	February	*March*	*April*
R. A. ...	11 h 28 m.	11 h 25 m.	11 h 22 m.
Decln. ...	4° 43′ N.	4° 59′ N.	5° 19′ N.

In Monthly Notices Roy. Astro. Soc. Vol. 97 the period of the binary 0Σ79 has been calculated to be 89·2 years. The semi-major axis of the orbit 0′·53 and its inclination ± 51°·6. The dynamical parallax is 0″·021 and the spectroscopic parallax 0″·022, while the trigonometric parallax is 0″·046.

Messrs. H. W. Newton and H. J. Barton have concluded in a comprehensive paper contributed to the above volume that Radio fadings are due to a solar agency travelling with the approximate speed of light from a limited part of the solar chromosphere.

New Minor planet close to the earth.

Herr K. Reinmuth Konigstubb discovered an object of the tenth magnitude on October 28 at R. A. 1 h.-34·2 m, and N. Decl. 8°-6′. Its perihelion distance is about 0·6, and its closest approach to the earth was in October and it was then less than 700000 miles from us.

H. SUBRAMANI IYER,
Govt. Astronomer.

Trivandrum.

GLEANINGS

67. The man who cannot wonder, who does not habitually wonder (and worship), were he President of innumerable Royal Societies, and carried the whole *Méchanique Céleste* and Hegel's *Philosophy*, and the epitome of all Laboratories and Observatories with their results, in his single head—is but a Pair of Spectacles behind which there is no Eye. Let those who have eyes look through him, then he may be useful.

CARLYLE in *Sartor Resartus*.

68. What I prophesied two and twenty years ago, as soon as I discovered the five solids among the heavenly orbits—what I firmly believed long before I had seen Ptolemy's *Harmonies*—what I had promised my friends in the title of this book, which I named before I was sure of my discovery—what 16 years ago I urged as a thing to be sought—that for which I joined Tycho Brahe, for which I settled in Prague, for which I have devoted the best part of my life to astronomical contemplations, at length I have brought to light, and recognised its truth beyond my most sanguine expectations. It is not 10 months since I first got the first glimpse of light, 3 months since the dawn, very few days since the unveiled sun, most admirable to gaze upon, burst on me. Nothing holds me; I will indulge my sacred fury; I will triumph over mankind by the honest confession that I have stolen the golden vases of the Egyptians to build up a tabernacle for my God far away from the confines of Egypt. If you forgive me I rejoice; if you are angry I can bear it; the die is cast, the book is written, to be read either now or by posterity, I care not which; it may well wait a century for a reader as God has waited six thousand years for an observer.

Kepler's Rhapsody after the discovery of the last of his "Laws of planetary motion".

MATHEMATICS EXPOSITION AT CENTRAL COLLEGE, BANGALORE

In connection with the College Day Celebrations of the Central College, Bangalore, on the 23rd Dec. 1937, presided over by the Hon. Mr. C. Rajagopalachari, Premier of Madras, A MATHEMATICS EXPOSITION was held introducing some interesting features and a short account of the same is appended herewith :—

The Exposition was divided into two sections devoted to Pure and Applied Mathematics and housed in two separate rooms.

The Pure Mathematics room had at the entrance two huge collapsible models of an ellipsoid and a hyperboloid of one sheet showing the circular sections (as in Hilbert-Cohn Vossen's *Anschauliche Geometrie*) cut out of thick cardboard. Right at the entrance and inside the room hung the letter π, the value to 707 places of decimals of which written on thick cardboard rolls was carried right round the room on the top of the walls in a prominent position and the roll of cardboard was illuminated all the way with coloured bulbs. The exhibits were classified into the divisions General, Algebra, Analysis, Geometry and Topology. The bound volumes of the *Encyklopadie der Math. Wissenschaften* as well as the *Principia Mathematica* were put up as exhibits. Among instruments and aids to calculation were put up logarithmic and other tables, a page from Jahnke u. Emnde's *Funktionen-Tafeln*, slide rules, planimeters, pantographs, elliptic compasses, parabolic curves, logarithmic scales, graphs illustrating numerical solutions of algebraic and differential equations, a Calculating machine, and photographs of integraphs and Coradis Harmonic analysator. Under recreations came some striking magic squares including the magic square of the knight move in chess, the Acht Damen Probleme, the Konigsburg bridge problem. Also a game of marbles depending on the binary scale of notation was arranged to be played between the visitors to the exhibition and the volunteer in charge. The famous problems of the theory of numbers were strikingly exhibited on specially prepared boards and included Fermat's last problem, Waring's problem, Goldbach's theorem with Vinagradow's partial solution and several others. An actual comb from out of a beehive was exhibited with a reading lens accompanying it so as to bring out the famous problem of maxima and minima associated with it. Coloured pictures representing hexagonal groups, the icosahedral group, crystal molecules and space partitioning were put up in the section on group theory. A historical development of geometrical notions was brought out in several specially prepared explanatory accounts written out in bold characters on drawing sheets pinned on to drawing boards. Models of the fine regular polyhedra, models illustrating conic sections, models of roulettes, and the helix were exhibited. Smaller collapsible models of conicoids (ellipsoid, hyperboloids, paraboloids cone, cylinder) showing circular sections were also put up. A number of plane configurations like the Pascal Hexagram etc. were illustrated by drawings. Space configurations like the Desargue's and Schlafli's Double Six were illustrated by large size wire models which were constructed for the occasion on the lines suggested in Hilbert-Cohn Vossen's book. The

geometry of the triangle was illustrated by complicated figures showing the important points, lines and circles associated with the triangle. Two large size drawing sheets contained blue prints of some fifty algebraic and transcendental plane curves obtained from Loria's book. Soap bubble models of minimal surfaces, the helicoid and the catenoid, were very popular exhibits and were highly appreciated.

In the section of topology were wire models of several knots illustrating Gordian cutting (as in Wendt. *Mathematische Zeitschrift* Bd. 42, Heft. 5), Pannwitz invariant and Dehn's Lemma. Five drawings illustrating the Four-colour problem and Borsuk's problem (picture in Kerekjarto's *Topologie*) were other prominent features in this section. An account of Riemannian and non-Riemannian Geometry concluded the first part of the exhibition, and at the exit was placed a large size black-board with complicated formulae written on it relating to mathematical logic, Tensor theory, continued fractions and analytic number theory etc. The walls of the room were adorned with enlargements mounted and framed, of Ramanujan, Cauchy, Riemann, Abel, Gauss, Weierstrass, Kronecker, Hilbert, Minkowski, Bois Reymond, Dirichlet and Cantor. A number of placards containing famous sayings like Plato's "Let no one who is ignorant of geometry enter here" Kronecker's "God made integers! all else is the work of man" etc. were nailed to the walls.

In the Applied Mathematics section, the exhibits were arranged under Mechanics, Astronomy and Astrophysics, Relativity and Quantum Theory. Exhibits were chosen with a view to illustrate some specific recent development rather than with an idea of comprehensiveness. In the Mechanics section were found two gyroscopes, several types of linkages and parallel motion, placards illustrating the Hamilton-Jacobi methods of analytical mechanics and the three-body problem. In the hydrodynamics section were fine photographs of flow round obstacles and Karman-street of vortices. Actual vortices formed by drops of mercuric chloride solution falling into potassium iodide solution were shown, and this experiment was universally popular. Models to illustrate the Prandtl-Karman theory of airfoils and the principles of ærodynamics were also exhibited.

The Astronomy section was subdivided into Spherical Astronomy, Solar System and Astrophysics. The first division had the usual celestial globes with explanations, large coloured star maps, the Nautical Almanac, a placard illustrating principles of navigation, a time-chart, some sheets of field Astronomy work containing actual results of altitude measurements of the Sun obtained by using the Sextant, and photographs relating to an imaginary trip to the Moon on a rocket as an illustration of astro-nautics. The members of the solar system were shown by means of coloured photographs fixed to several drawing boards and prominently exhibited with suitable explanations. Three models illustrating the occurrence of eclipses, phases of the moon, the planets with their satellites going round the sun were also shown. There were also other photographs of stellar objects like star clusters, galactic and extra-galactic nebulae, variable stars, novae. In the astrophysics section were some specially prepared charts showing graphically the development of Astronomy for the last 100 years on the basis of the numbers of radial velocities, proper motions, parallaxes etc. known at each epoch. Methods of stellar statistics and stellar photographs were put in evidence by means of graphs, photographs and explanations. The Hertzsprung-Russel Diagram and Eddington's Mass-Luminosity relation were

shown on large scale drawings. The evolution of giant, main-sequence and dwarf stars according to Russel were shown diagrammatically. A very striking exhibit was a model of the giant star Antares cut out of thick cardboard as a circular disc of nearly 4 ft. diameter hung along with a small disc of ½ in. radius to denote the size of the Sun. The drawing was done to scale to represent the actual diameter of 143 million kms. of the star. Pictures of the several spectra, variable star curves and a graph showing the Leawilt-Shapley relation were included in this section.

The instruments exhibited in the room were a theodolite with solar attachment, a sextant with artificial horizon, a 3 inch completely equipped equatorial and a siderial clock. Outside the building and in the open verandah, the 6 in. equatorial telescope of the Department was set up and visitors to the exhibition were allowed to look through during the several nights at celestial objects like Jupiter's moons, Saturn's rings, Mars, the Nebula in Orion, the Nebula in Andromeda, the double star of β-Cygni, and several other interesting objects. This drew a huge number of visitors and admission had to be by tickets.

In the Relativity and Quantum Theory section a large number of specially prepared explanatory charts were exhibited showing the principal results of the theories. In one was collected the fundamental formulæ like

$$E=mc^2, \quad t=h\nu, \quad pq-qp=ih/2\pi\,; \quad \alpha=\frac{2\pi e^2}{ch}; \quad a=\frac{h^2}{4\pi^2me^2}\,; \quad \lambda=\frac{h}{mv}$$

In another were shown the important matrices denoting the co-ordinate and momentum for the harmonic oscillator, the Pauli matrices, Dirac matrices, and the operators of the neutrino-theory of light. Short explanations of Schrodinger's wave-equation and its mathematical relationships were given on one sheet. Crystal models to illustrate applications of quantum theory to the lattice theory of solids were also exhibited. An explanatory introduction was afforded to the fundamental particles of nature like electron, proton, positron, negative proton (?), neutron, photon, neutrino and the α-particle.

Stereoscopic drawings, with a stereoscope accompanying, were shown of two dimensional analogues of elliptic and hyperbolic spaces, and the projective geometry in them as illustrations of the principles of Relativity theory. There were also some explanatory introductions to several problems of this theory.

The Applied Mathematics room also contained several enlarged photographs including those of Lagrange, Laplace, Kepler, Galileo, Newton, Poincaré, Minkowski, Einstein, Born, Kapetyn, Newcomb, and Shapley. Placards containing famous mathematical sayings and black-boards with formulæ from Mathematical Physics (Partial Diff. Eqns.), Relativity and Quantum theory were put in suitable places.

The room adjoining the Applied Mathematics Hall was converted into a Dark Room for showing slides to the visitors and a large number of Astronomical slides dealing with recent di coveries were shown and explanation given by the volunteers in charge.

The Exposition was arranged by B. S. Madhava Rao with the active co-operation of K. Venkatachala Iyengar and other members of the Mathematics Department.

Indian Statistical Conference

FIRST SESSION, CALCUTTA, 1938

PRESIDENTIAL ADDRESS BY PROFESSOR R. A. FISHER, SC.D., F.R.S.

In the original sense of the word 'Statistics' was the science of Statecraft; to the political arithmetician of the eighteenth century, its function was to be the eyes and ears of the central government. It could tell the Prince how many able-bodied men he might mobilise, how many would be needed for the essentials of civil life; how numerous or how wealthy, were sectarian minorities who might resent some contemplated change in the laws of property, or of marriage; what was the taxable capacity of a province, his own, or his neighbours.

This aspect of statistical information seems to have been long neglected in England, and we should have to look as far as Italy for a centralized statistical service capable of ensuring that Il Duce is the best informed potentate in Europe. It is certain that the power which such knowledge gives, like the power of lethal weapons, may be abused. On the other hand it *is* in the public interest that statesmen responsible for political decisions should be able, so clearly as is possible to foresee the consequences of their public actions.

In democratic countries the emphasis has long fallen upon the important task of providing public information. This function of official statistics is of value both for the aid it gives to private enterprise to use its initiative wisely, and for allowing public opinion an opportunity, at least, of acquiring such a knowledge of quantitative facts as will enable it to pursue aims which are not only worthy, or desirable, but are also within the bounds of practical possibility. It enables public opinion, *if it will*, to size up its own problems. The Socratic dictum "know thyself" is applicable even more to peoples than to individuals.

Statistical work is also largely employed as a means to internal efficiency in each great department of Government. I need not stress this aspect of official statistics, which is not I believe commonly neglected; save to point out that here also the official service requires a supply of men, who, if they are to be competent should have at least some specialised statistical training.

More recently, the great centres of scientific research have discovered that statistical science has a vital part to play in all quantitative problems. I will later give some account of this development, which has revolutionized the technical methods of statistics, and has greatly extended the public functions it can fulfil. Heads of research institutions in all countries are feeling the advantage of having as right-hand men statisticians technically capable of drawing up a research programme or of applying to a projected research a rational and objective criticism.

I have formed an opinion, which, be it right or wrong, I should like you to bear in mind throughout this address, that statistics in England has suffered severely from the wide separation, due to our long political history, which has grown up between official and academic statistics; or, to speak functionally between the duties of collection, enumeration, tabulation and publication, which absorb the time of official statisticians, and the duty of study, analysis and interpretation which falls to the lot of the mathematical or theoretical statisticians. A body such as the Royal Statistical Society does something to bridge this gap.

How wide the gap may seem may be illustrated by some witty comments made by A. L. Bowley in criticism of the presentation of official data. He was moved to draft an ideal footnote, which might find a place in any government publication. I cannot recall his exact words but the following is an attempt to recapture the spirit of his criticism.

Footnote:— (*a*) The terms used in the headings and margins of the table are all employed in a technical sense, known only to officers who compiled it and which they are unable for official reasons to divulge.

(*b*) The sub-divisions of the table and the region to which it refers have been changed since the last return published.

(*c*) Before tabulation the data have been subjected to numerous adjustments, allowances and other corrections, of a kind to vitiate any tests of significance which the reader may be tempted to apply to them.

The academic mind, as we know, is sometimes capable of assuming an aggressive attitude. The official mind, on the contrary, is and has to be, expert in the art of self defence. If half of Bowley's indictment is true, it will always be easy when analytic methods are rashly applied to official material, to show how frequently the material falsifies the assumptions made in the course of analysis: to sneer at the elaboration of the calculations employed, or at the refinements which have been introduced, always without emphasizing the obvious fact that the more imperfectly the work of compilation has been done the more elaborate and difficult must be the task of extracting objective information. My point here is that in developing her statistical services India might learn from the difficulties which England had encountered, and somehow contrive neither to allow official statisticians to be blinded by ignorance of method, nor to allow academic statisticians to be sterilized by lack of responsible experience.

So recently as twenty years ago the task of the statistician, faced with an accumulated mass of observational data was understood to consist in the calculations of certain average values; by that time also it had become usual to employ the quantities of the second degree, squares and products, to elicit from the data what are known as "probable errors," regarded as appropriate to the average values obtained. In putting the matter in this way I mean a wide interpretation to be given to the term 'average' for these are of many different kinds and any large body of data will yield an immense variety of them. Which to choose must depend partly on what sort of information it is desired to obtain, and this part is not properly a statistical question. But expert statistical knowledge was recognized to be necessary for a comprehension of the

arithmetical processes sufficient to ensure that they were appropriate to the meanings to be placed on them. This narrow measure of responsibility was however enlarged by the fact that all extensive bodies of data are liable to contain information on points which were not in view when they were collected; to recognize such information, and to find the means of eliciting it was most stimulating part of the statisticians' task.

As an illustration I may point to the widespread and important type of data known as time-series, such as are provided by annual figures of births, deaths, exports, prices and the whole range of vital and commercial statistics, the proper treatment of which has been a classical problem to all who are interested in national or sociological questions. Data of the same kind are amassed by meteorologists, and arise when agricultural experimentation is carried out continuously on the same land. In studying such data as this at Rothamsted farm it was found that a series of averages, using weights determined by position in the sequence, would provide, *first*, the linear trend representing the greater part of soil deterioration, *next*, a few terms representing other slow changes ascribable to changes in variety, cultural practice and intensity of weed infestation, and *finally*, a residuum of more rapid fluctuations dominated by the variation of the weather from season to season. Moreover, the separation effected in this way was such as to allow the year to year fluctuation to be studied in relation to meteorological causes, just as if the series were not affected at all by the large slow changes actually present.

The classical theory of errors due to the great German mathematician Gauss had shown how the interpretation of such estimates could be aided by taking account of the whole body of discrepancies. The residue of unexplained variation provide, in fact, the material for the calculation of probable errors, or in more modern usage of Standard errors. In 1908 the late W. S. Gossett, whose untimely death last year still casts a shadow on our discussions, wrote under the pseudo-nym of "Student" a paper, little appreciated at the time, which opened the door to a refinement of exactitude of which the theory of errors had hitherto seemed incapable. The concept of standard error is in fact insufficiently exact for application without reserve to the small samples of observations which the experimenter can command. Nevertheless, it was shown possible in an important range of cases to develop rigorously exact tests of significance from which the concept of standard error may be entirely eliminated. That the importance of Student's suggestion was not at once recognized illustrates all too clearly how ignorant academic statisticians were of the important decisions which experimental scientists, like Statesmen, must take, always on the basis of a limited knowledge of fact. Biological research, however, throughout the world bristled with problems which required exact tests adapted to a very wide variety of logical situations and in due course such tests came into existence. In a few years more they were in general use among experimentalists. The period which followed has shown the somewhat ludicrous spectacle of entomologists, foresters, plant physiologists and others with no trace of mathematical pretensions, applying freely and with understanding in their daily work mathematical refinements, which most official statisticians could not understand, and which too many teachers in mathematical departments were unable to expound.

The development of exact test of significance had been based on the mathematical solutions of a number of what are known as *problems of distribution*. The solutions found were not very difficult mathematically. Certainly they were not beyond the power of the great series of mathematicians who, from the time of Laplace, have given their minds to the theory of probability. Had these minds been put into direct contact with the problems which beset the Laboratory worker we cannot doubt that they would have been solved by them some sixty years earlier. I want to insist on the important moral that responsibility for the teaching of statistical methods in our universities must be entrusted, certainly to highly trained mathematicians, but only to such mathematicians as have had sufficiently prolonged experience of practical research, and of responsibility for drawing conclusions from actual data, upon which practical action is to be taken. Mathematical acuteness alone is not enough. My revered teacher Prof. Whitehead of Cambridge used to say in one of his courses: "The essence of applied mathematics is to know what to ignore." And when I read current publications in mathematical statistics I am continually and forcibly reminded of the wisdom of this remark.

The solution of problems of distribution, although stimulated by the practical needs of chemists and biologists, had at once important mathematical consequences. For, if the statistician asks himself, as he ought, "What is my best method of averaging or combining the data before me," so as to elicit information on some definite issue, it is evident that he must be guided by the magnitude, and the nature of the sampling errors to which different estimates are liable. This, in a few words, is the genesis of the theory of estimation. In this theory we discuss what advantages each of innumerable possible estimates possesses, and what procedure will enable us to combine these advantages in our estimates. I shall hope to develop the theory more fully in a series of lectures at the University of Calcutta next week. For the moment, I want to note only the consequences of the progress so far made in this field as they affect the future of statistical research, and the position and responsibilities of the statistician.

A few years ago no statistician who had completed his work on an extensive mass of data could feel any rational assurance that some other statistician, using the same data, could not by some new method calculate estimates much more precise than his own. In these circumstances it was natural that if his results were actually rather precise he should display them with some satisfaction, if not he might in his chagrin even suppress them. High correlations, and strongly significant comparisons were displayed with considerable pride, for they seemed to indicate not merely that the data were good, but that they had been brilliantly handled. But now, observe the change, so soon as any young graduate who has mastered the theory can lay down the equations for the most efficient estimates possible, the whole tone of the subject has been altered. The statistician is no longer an alchemist expected to produce gold from any worthless material offered him. He is more like a chemist capable of assaying exactly how much of value it contains, and capable also of extracting this amount, and no more. In these circumstances it would be foolish to commend a statistician because his results are precise, or to reprove because they are not. If he is competent in his craft, the value of the result follows solely from the value of the material given him. It contains

so much information and no more. His job is only to produce what it contains.

The mathematical measurement of quantity of information has another and more fruitful consequence. If we want more than has been found to be present we are no longer tempted to harass the statistician to work miracles. On the contrary, since he can evaluate the material, we may require him to explain what was wrong with the data that more did not come of it. What supplementary information which could also have been collected would have been worth collecting? At what points could the material be most profitably amplified? In what respects also has the labour of collection been unprofitable? So that our resources may be employed in future where the yield is highest. Get the statistician to redesign the enquiry, and to justify his new plan by the methods of costings accountancy. Here the door is opened on work very well worth doing indeed. Immensely laborious calculations on inferior data may increase the yield from 95 to 100 per cent. A gain of 5 per cent of a perhaps small total. A competent overhauling of the process of collection, or of the experimental design, may often increase the yield ten- or twelvefold, for the same cost in time and labour. To consult the statistician after an experiment is finished is often merely to ask him to conduct a *post mortem* examination. He can perhaps say what the experiment died of. To utilise this kind of experience he must be induced to use his imagination, and to foresee in advance the difficulties and uncertainties with which, if they are not foreseen his investigations will be beset.

I have tried to show that India, in the construction of her national organization, has a great opportunity to utilise the newly discovered possibilities of Statistical Science. This Conference is itself the strongest evidence that the will to seize this opportunity is not lacking. When I see too the widespread interest developing in this country in all branches of statistical work, and above all the brilliant school of workers that Prof. Mahalanobis has gathered round him in this University I suggest that we can put every confidence that the work will be carried forward with the intellectual integrity that such a task requires.

THE SILVER JUBILEE SESSION OF THE INDIAN SCIENCE CONGRESS

The Indian Science Congress met at Calcutta between the 3rd and 9th January 1938 amid scenes of unparalleled enthusiasm to celebrate its Silver Jubilee Session, the importance of the occasion being heightened by the presence of a fairly large number of distinguished British and Foreign savants who actively participated in the proceedings. The occasion attracted nearly 1600 delegates while the total number of papers communicated was more than 800 distributed among 13 sections.

The Viceroy who opened the session welcomed the participation of British Delegates as a happy augury for the close co-operation between India and the Scientific world outside, and made a fervent appeal for the application of scientific knowledge for the betterment of the physical and economic condition of the masses. Sir James Jeans, President of the session,

in a brief introductory speech paid a glowing tribute to the scientific achievements of the late Lord Rutherford and the late Sir J. C. Bose and then read the Presidential address prepared by Lord Rutherford. The Address dealt with the contributions which India had made in the field of pure and applied science both by individual workers and by organised bodies like the Indian Cotton Committee and the Agricultural Research Council and with the needs of the future. The personal touch came with an account of modern work on the transmutation of matter and the discovery of the neutron.

The sessions split into sections which met every morning from 9-30 A.M. to 12 noon. In the evening from 1 P.M. to 4-30 P.M. there were usually joint discussions on special topics and thereafter Garden parties and excursions. The day wound up with a popular lecture by an eminent scientist. The excursions included a river trip on the Hooghly, a visit to the Bengal chemical and pharmaceutical works and the Bengal Immunity Company. One of the most interesting experiences was that of witnessing a Bratachari dance by G. S. Dutt Esq., its founder President and his disciples,—a series of rythmic movements of great vigour and significance in which the resurgent spirit of Young Bengal is seeking to express itself.

We have great pleasure in publishing a portrait of Sir Arthur Eddington who was one of the most eagerly sought for among the British Delegates.

THE FIRST INDIAN STATISTICAL CONFERENCE

was opened by His Excellency the Governor of Bengal on 7th January 1938. We have great pleasure in reproducing elsewhere the Presidential Address delivered by Prof. A. Fisher, successor to Karl Pearson in the Galton chair of Eugenics, London University, a copy of which was kindly supplied by Prof. Mahalanobis. There was a large number of papers dealing with theoretical Statistics and some of its concrete applications. The success of the Conference is due in no small measure to the energy and enthusiasm of the band of researchers working in the Statistical Laboratory at Calcutta under the guidance of Prof. Mahalanobis.

GLEANINGS

69. "The essence of Applied Mathematics is to know what to ignore."

WHITEHEAD

(from Prof. Fisher's *Presiden'ial Address at the Calcutta Statistical Conference.*)

"A positive result may be bought at too great a cost; for it may be due to an abstraction which neglects the 'sting' of the problem".

JOACHIM
The Nature of Truth

REVIEWS

W. GONTCHAROFF, *Détermination des fonctions entières par interpolation*

Actualités Scietifiques et Industrielles, No. 451, price 12 Francs.

Let $F(z)$ be an integral function and let

$$a_{nm} \begin{pmatrix} n=0, 1, 2, \ldots \\ m=0, 1, \ldots, n \end{pmatrix} \quad \cdots\cdots \quad (1)$$

be a triangular array of points in the complex plane. There is a unique polynomial* $P_n(z)$ of degree n satisfying the conditions

$$P_n(a_{nm}) = f(a_{nm}), \qquad (m=0, 1, \ldots, n) \quad \cdots\cdots \quad (2)$$

There is also a unique polynomial $Q_n(z)$ of degree n for which

$$Q_n^{(m)}(a_{nm}) = f^{(m)}(a_{nm}), \ (m=0, 1, \ldots, n) \quad \cdots\cdots \quad (3)$$

When a_{nm} is independent of the first index we get a linear array or sequence

$$a_m, \ (m=0, 1, 2, \ldots) \quad \cdots\cdots \quad (4)$$

What are the conditions on the array (1) or (4) and the function $F(z)$ which will ensure the uniform convergence, in *any* finite circle $|z| \leqslant R$, of the corresponding polynomials $P_n(z)$ and $Q_n(z)$ to the limit function $f(z)$? This is the main problem discussed in the pamphlet under review. After stating the problem under a slightly more general form and after obtaining explicit formulæ for $P_n(z)$ and $Q_n(z)$ for the cases (1) and (4) in chapter I, the author discusses in chapter II, two cases of (2) given by

$$\text{(i)} \quad |a_{nm}| \leqslant \xi_m, \ \xi_m \sim \omega m^{1/\rho} \quad \text{and} \quad \text{(ii)} \quad a_m = \omega m^{1/\rho}$$

in detail. In chapter III the problem corresponding to (3) for the array (4) is considered at some length and the author concludes with a discussion of some questions allied to the subject-matter of the pamphlet. There is a well-chosen bibliography at the end bearing on the questions discussed in the pamphlet.

V. G. IYER.

G. BOURION, *L'ultraconvergence dans les séries de Taylor*, Actualités Scientifiques et Industrielles, No. 472, price 12 francs.

This booklet is devoted to the study of the phenomenon called 'over-convergence' (ultraconvergence) in connection with power (Taylor) series. Let $S_n(x)$ be the sum to first $(n+1)$ terms of the Taylor series

$$\sum_{n=0}^{\infty} a_n x^n, \quad \cdots \quad (1)$$

which is supposed to have a finite radius of convergence which we can suppose to be unity without loss of generality. Then

$$\lim_{n\to\infty} S_n(x) = F(x), \ |x| < 1, \quad \cdots \quad (2)$$

* Some of the points a_{nm} $(m=0, 1, \ldots, n)$ might occur more than once in the same row. We say that $F(z) = \phi(z)$ at $z=a$ counted ρ times if $f(a)=\phi(a)$, $f'(a)=\phi'(a) \ldots$ $f^{(\rho-1)}(a) = \phi^{(\rho-1)}(a)$

where F (x) is the function defined by the element (1) while the limit on the right side of (2) cannot exist for *any* x such that $|x|>1$. But it can happen that a subsequence $\{ S_{n_k}(x) \}$ of $\{ S_n(x) \}$ converges for points outside $|x| \leqslant 1$ in which case the sequence $\{ S_{n_k} \}$ is said to *over-converge* outside the circle of convergence $|x|<1$ of (1). The series (1) is said to be a gap series (série á structure la cunaire) if it can be writen in the form

$$\sum_{n=0}^{\infty} \alpha_n x^n + \sum_{n=0}^{\infty} \beta_n x^n, \quad \alpha_n + \beta_n = a_n, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (3)$$

where $\sum_{n=0}^{\infty} \beta_x x^n$ has a radius of convergence greater than unity while

$$\alpha_n = 0, \quad m_k \leqslant n \leqslant n_k \, (k=1, 2, \ldots), \quad m_k < \lambda \, n_k, \quad \cdot \quad \cdot \quad (4)$$

where $\lambda < 1$ is fixed and in this case the relative length $\frac{n_k - m_k}{n_k}$ of the gap is said to bounded below since it is greater than $\left(\frac{1}{\lambda} - 1\right) > 0$. One main result regarding over-convergence is that in the case of gap series the sub-sequence $\{ S_{n_k}(x) \}$ determined by the gaps (4) converges in a sufficiently small circle round any point x_0 on the circle of convergence $|x|=1$ where the function $F(x)$ given by (2) is regular. The book gives a detailed discussion of this and allied results in chapters I and II.

The polynomial $S_n(x)$ vanishes at n points. Consider the set S of the totality of these points for $n=1, 2, \ldots$ A theorem due to Jentzsch says that every point of $|x|=1$ is a limit point of the set S. But if we consider a subsequence $\{ S_{n_k}(x) \}$ and the corresponding set of points where these polynomials vanish, there can be points on $|x|=1$ which are not points of accumulation of the set in question. There exists an intimate relation between this question and the problem of over-convergence. In chapter III and the succeeding chapters this connection and other allied questions are considered at some length. There is a good bibliography at the end bearing on the problem of over-convergence and its relation to the points of accumulation of the set of zeroes of the polynomial sections of the Taylor series (1).

V. G. IYER.

E J. GUMBEL, *La durée extréme de la vie humaine,* Actualités Scientifiques et Industrielles, 520; Statistique Mathématique II, Hermann, Paris, 1937, Pp. 63, 18 francs.

This is the second tract in the section 'Statistique Mathematique' of the general series 'Actualités Scientifiques et Industrielles', the first one 'L'Emploi des Observations Statisques—Méthods d'Estimation' by the editor Georges Darmois having appeared in No. 356 in the year 1936. Though these two tracts are published in two successive years, the enormous difference between their numbers gives a good measure of the progress in various Sciences, which is being recorded in this series.

The present tract by E. J. Gumbel of the Faculté des Sciences, Lyon, is a condensed and lucid presentation of the novel ideas evolved by the author

himself between 1932 and 1936 concerning the statistical distributions of the highest ages at death in a population. The method followed is based on the researches of R. A. Fisher on the limiting forms of the frequency distribution of the largest or smallest member of a sample. It is a fruitful one, which the author has applied to various other studies also, e. g., floods and extreme distances between radio-active emissions.

The substance of the exposition is spread over *five* chapters and comprises the Tables of Mortality, Theory of the last age, the Distributions of the greatest age in Switzerland and Sweden, the formula of Lexis and American observations, and the formula of Gompertz and a historical account of the greatest age in Sweden. The fundamental formula round which the whole text turns with all its applications to the well-known hypotheses of mortality due to Lexis and Gompertz is

$$W(x) = \mu(\bar{\omega})\, e^{-y-e^{-y}}$$

where $y = \mu(\bar{\omega})(x-\bar{\omega})$, $\bar{\omega}$ is the maximum value of $d\, W(x)/dx$, called the last age (dernier âge) and $\mu(x)$ the intensity of mortality at age x.

The theory of 'Dernier Age' involves an interesting paradox:

> A 'défavourable' table (such as the Hindu life-table in which the normal age is as low as 38) shows an enormous superiority in the point of longevity to the 'favourable' table (such as that of the Australians), when the number of observations is large.

The 'dernier age virtuel' of Hindus varies from 103·59 to 130·39 as against that of Australians varying from 98·23 to 108·15, for the number of observations ranging from 10^3 to 10^8 (p. *vide* 57).

While concluding with the remark that the arguments are in a sense parallel to those of Bortkiewicz in his 'Laws of small numbers', the author humorously adds that just as Bortkiewicz's study of the number of soldiers killed per annum by a horse does not at all increase our knowledge of horses, even so, the study of the destinies of *Mathusaleme* shows nothing more than that the extreme values follow certain rules of the Calculus of Probability and can be verified by observations, provided these are trustworthy and the theory sound. However, there is no doubt that the theory will be of great interest to students of Actuarial Science. Any revision scheme for Government Pensioners must take into consideration this new theory.

A. A. KRISHNASWAMI IYENGAR.

LAVRENTIEFF *Sur les fonctions d'une variable complexe représentables par des series de polynomes*, (Actualités Scientifiques et Industrielles, No. 441); price 15 Fr.

A famous theorem of Weierstrass states that, given a continuous function F (x) in an interval (a, b), there exists a sequence of polynomials $[P_n(x)]$ which converges uniformly to F (x) in (a, b). Conversely the limit of a uniformly convergent sequence of polynomials is a continuous function. In the memoir under review, the author collects together all the important results available regarding the question of what happens when the condition that $P_n(x)$ converges

uniformly is dropped and when an interval is replaced by a curve or a domain in the complex plane. The fundamental result in this connection is due to Montel who proved that if a sequence of polynomials converges at *every point* of a domain, it converges *uniformly* in the domain except for a perfect non-dense set. These points of non-uniform convergence are said to be irregular points. It follows that the limit function should be holomorphic in the domain except may be at the irregular points. The structure of the set of irregular points is investigated in detail in the book under review and the behaviour of the function $f(z)$ at the points where it is not holomorphic in the domain is determined in order that it may be the limit of a sequence of polynomials in the given domain.

BOOKS RECEIVED FOR REVIEW

L. V. GURJAR: *Theory of Elasticity*, Arya-Bhushan Press, pp. 108, 1937, Price Rs. 4.

MAHAJANI, G. S.: *The application of moving axes methods to the Geometry of Curves and Surfaces*, pp. 60, 1937, Arya-Bhushan Press, Poona.

Actualités Scientifiques et Industrielles, HERMAN et Cie, Paris.

No. 450 KARAMATA: *Sur les théorèms inverses des procedes de Sommabilité*, 45 pp. Price 12 Frs., *1937*.

No. 465 GONTCHAROFF, W.: *Determination des fonctions entiers par interpolation*, 48 pp.; 1937, 12 Frs.

No. 472 BOURION, G.: *L'Ultraconvergence dans les séries de Taylor*, 46 pp., 1937, 12 Frs.

No. 493 LAMBERT, R.: *Structure générale des nomogrammes et des systems nomographiques*, pp. 64, 1937, 15 Frs.

No. 503 ONICESCU, O et MIHOC, G.: *La dépendence statistique, Chaines et familles de chaines discontinues*, pp. 45, 1937, 15 Frs.

No. 518 NOETHER, E. und CAVAILLES, J.: *Briefwechsel Cantor-Dedekind O*, pp. 60, 1937, 20 Frs.

No. 520 GUMBEL: *La durée extreme de la vie humaine*, pp. 63 1937, 18 Frs.

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